A Detailed Analysis of Systematic Errors in the Atwood Machine Experiment: The Effect of the Pulley's Friction on the Uncertainty of the Measurement Results

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"It is better to be vaguely right than exactly wrong." This statement,[1] applied to measurements in a lab, highlights the fact that systematic errors in experimental measurements can bias the results away from the true values even though one repeats the measurements many times to reduce the effects of random errors. In this paper, I demonstrate experimentally how the frictional force of the pulley's bearing (FFPB) in the Atwood machine as a major source of systematic errors affects measurements of velocity and acceleration in the Atwood machine. I analyze the errors of measured quantities in a systematic manner according to the international standards available for evaluating and expressing the uncertainty of measurement results.[2]

Introduction

The Atwood machine provides a rich substrate for research into fundamental physics and related application designs[3–6] and has been used to explain Newton's first and second laws of motion in introductory lab courses. Error analysis of the experimental results is a key part of conclusions in lab reports. The effects of frictional forces such as the frictions of air and pulley are difficult to quantify in experiment and, therefore, usually are ignored in calculations and have not been considered in lab reports. Such simplifications cause the measured results to not agree with the theory. Specifically, because of FFPB in the Atwood machine,[7] the recorded data don't completely agree with the calculations performed with Newton's laws of motion. In this paper, I demonstrate how one can control the sources of errors in order to get values of the measured quantities in better agreement with the theoretical results obtained from the principle of inertia and from the relation between force and acceleration.

Equipment setup and extrapolating the FFPB

As shown in Fig. 1, a support column graduated in millimeters (1) is located on a base (2). By adjusting the base screws (3), we align the support column vertically with the help of leveler tool (4). The timer device (5) has two channels to receive signals. Channel A (6) is connected to time sensor 1 (7) located at location y_1, and channel B (8) is connected to time sensor 2 (9) located at location y_2. The pulley (10) of mass M_0 is installed on top of the support column, and a thin string (11) that passes over it is connected to mass 1 (12) and mass 2 (13), one at each end. These two masses are equal and are denoted by M. A light disk (LD) (14) of mass m_0 along with an overload (15) of mass m can be mounted on mass 1 and placed on the mechanical valve (16). If we pull the lever (17) of the mechanical valve slowly, then the valve will open and mass 1, along with the mounted LD and the overload, will start to move down from the rest position.

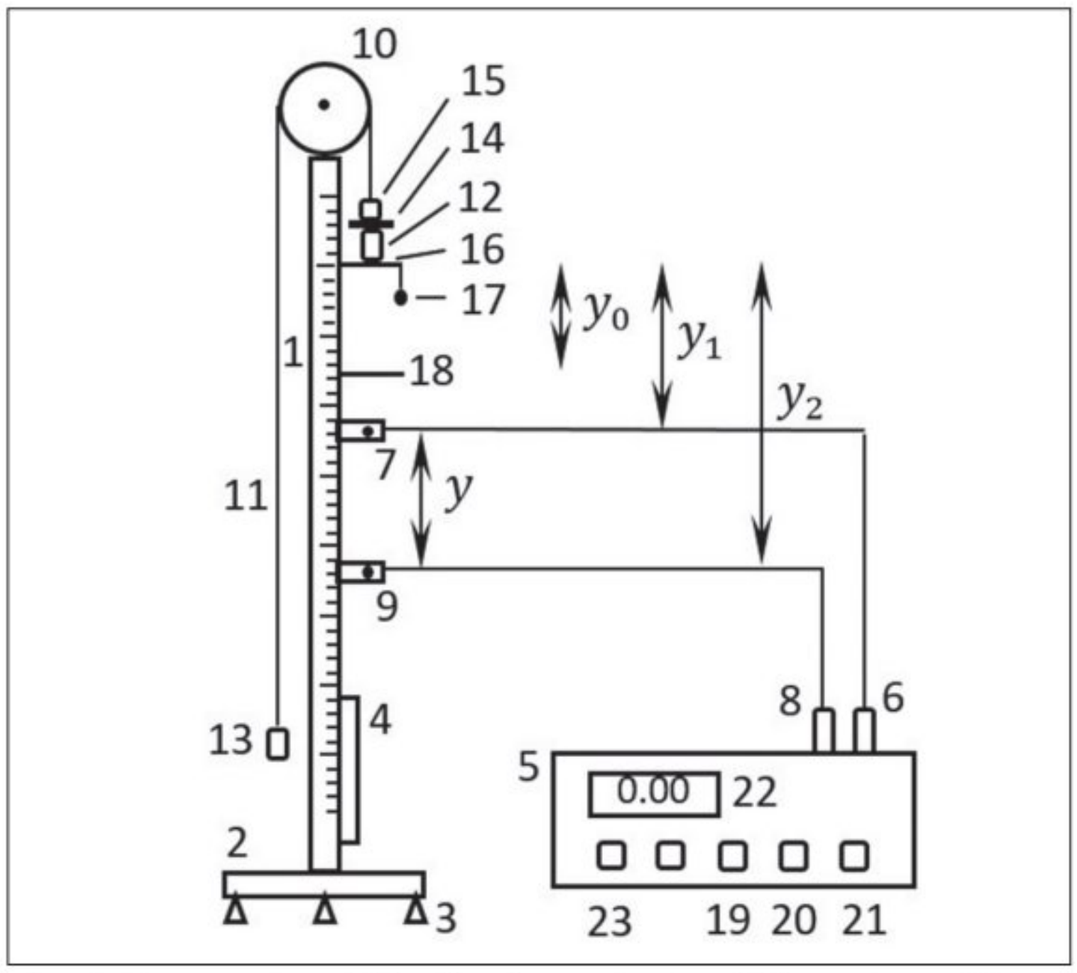

Fig. 1. An experimental setup for a standard Atwood machine.

It is noted that any start–stop timer could be used for this experiment (I use a memory time counter model J201-4C as the timer device). The time sensor used as a photogate in the present experiment is a simple IR sensor that consists of an IR LED and an IR black receiver photodiode. The receiver detects the IR radiation and converts it into an electrical signal that can be processed. The mechanical valve is a simple metal plate that is hinged to an axis. When the hinge is released by

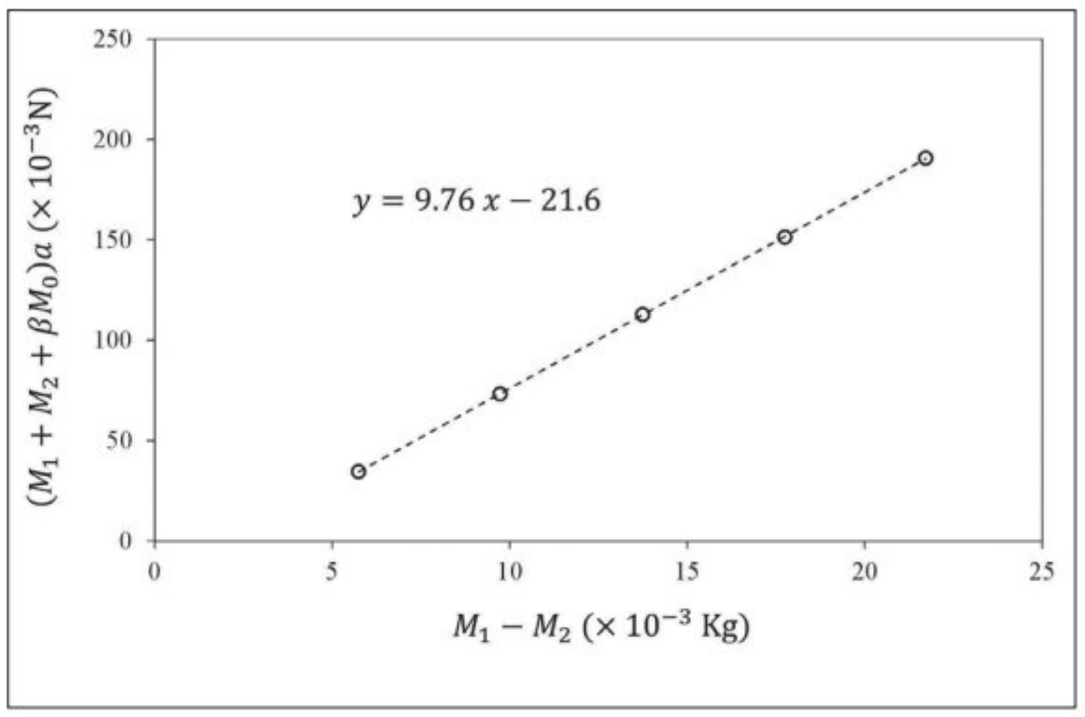

Fig. 2. Plot of $y = (M_1 + M_2 + \beta M_0)a$ in terms of $x = M_1 - M_2$. The total mass $M_1 + M_2$ is kept at constant value 155 g. We could choose other values $M_1 + M_2 \sim 10^2$ g that will not damage the machine. Results show that with such choices of $M_1 + M_2$, the extrapolated value of FFPB doesn't change significantly. The slope of the line can be compared with the theoretical value of the gravitational acceleration in the location of lab ($g = 9.797$ m/s^2 in this case).[10]

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Table I. Measured values of y and t and results for the speed v, the uncertainty Δv, and the relative uncertainty $\Delta v/v$ in the presence of the LD for the machine moving at a constant speed (see Section A2 of the Appendix[9] for calculations). The first row shows the run number i, and the total number of runs is 10.

i	1	2	3	4	5	6	7	8	9	10
y (m)	0.100	0.200	0.300	0.400	0.500	0.600	0.700	0.800	0.900	1.000
t (s)	0.174	0.357	0.526	0.710	0.873	1.081	1.235	1.409	1.569	1.767
v (m/s)	0.575	0.560	0.570	0.563	0.573	0.555	0.567	0.568	0.574	0.566
Δv (m/s)	0.007	0.004	0.003	0.002	0.002	0.002	0.002	0.001	0.001	0.001
$\Delta v/v$ (%)	2	0.6	0.4	0.3	0.3	0.2	0.2	0.2	0.2	0.2

pulling the lever, the metal plate falls due to its weight, allowing mass 1 to fall.

When the LD is used, when mass 1 passes through the separating ring (SR) (18), the LD and the overload on it are immediately separated from mass 1 (because the radius of the SR is smaller than the radius of the LD) and the machine continues to move at a constant speed v given by

$$v = \frac{y_2 - y_1}{t_2 - t_1} = \frac{y}{t}. \quad (1)$$

Here, mass 1 passes through time sensor 1 (2) located at y_1 (y_2) at time t_1 (t_2), and $t = t_2 - t_1$ is the time it takes mass 1 to travel the distance $y = y_2 - y_1$.

In the absence of the LD, channel A must be connected to the mechanical valve, so time sensor 1 is removed from the circuit as well as the SR, and we expect mass 1 along with the overload to move with a constant acceleration a with the equation of motion given by

$$y_2 = \frac{1}{2}at^2. \quad (2)$$

Here, t is the time it takes mass 1 to travel the distance y_2 (see Fig. 1).

If F denotes the FFPB, the acceleration of the machine in terms of the masses and the gravitational acceleration g is given as follows[8,9]:

$$a = \frac{(M_1 - M_2)g - F}{M_1 + M_2 + \beta M_0}. \quad (3)$$

I have used a plastic pulley with total mass $M_0 = 14.00$ g. The radius of the pulley is $R = 30$ mm and the rotational inertia of the pulley is $\beta M_0 R^2$ with $\beta = 1/2$. The data in Fig. 2 can be fitted by a straight line with the intercept 21.6×10^{-3} N, which represents an estimation of F. To obtain each point of the graph in Fig. 2, the acceleration a has been averaged over 10 values corresponding to different values of y_2 in Eq. (2).

Precision of measured quantities

Figure 3 shows the graph of y in terms of t in Eq. (1), with the values recorded in Table I. Points of the graph are located almost along a straight line, which indicates movement at a constant speed. The dashed line represents the best straight line that can fit the points of the graph. The slope of this line is 0.568 m/s.

The histogram in the inset of Fig. 3 shows how many times each measured value of v is repeated, and the dashed curve shows the normal distribution with mean value $\bar{v} = 0.567$ m/s and standard deviation $\sigma = 0.007$ m/s, which fits the histogram. Here, $n = 10$ is the total number of measurements, and the histogram tends to a normal distribution with increasing n. Therefore, we can consider the average value of uncertainties $\overline{\Delta v} = 0.002$ m/s as the final uncertainty in measurement of v, which is in agreement with the standard uncertainty $\bar{\sigma} = \sigma/\sqrt{n} = 0.002$ m/s. The result of $v = 0.567 \pm 0.002$ m/s shows the relative uncertainty in speed measurement equal to 0.4%, which is related to random errors. Therefore, the precision of measured speed in this experiment is 99.6%.

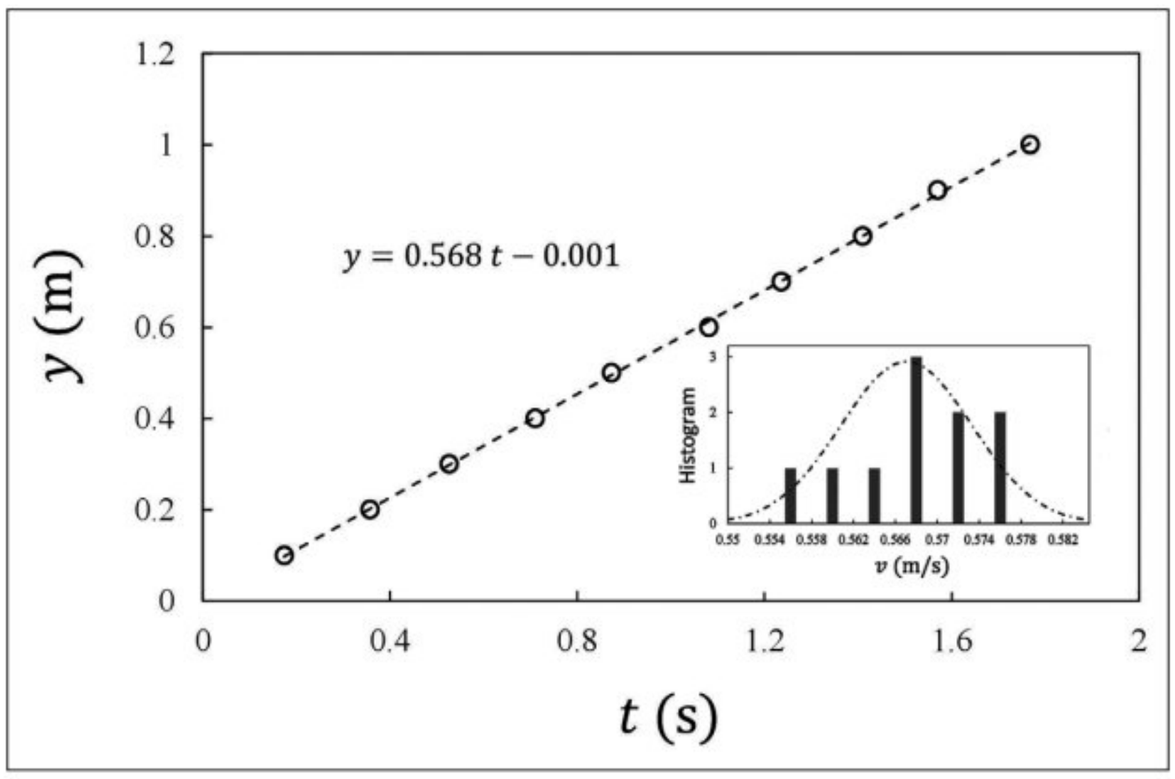

Fig. 3. Plot of y vs. t in the presence of the LD.

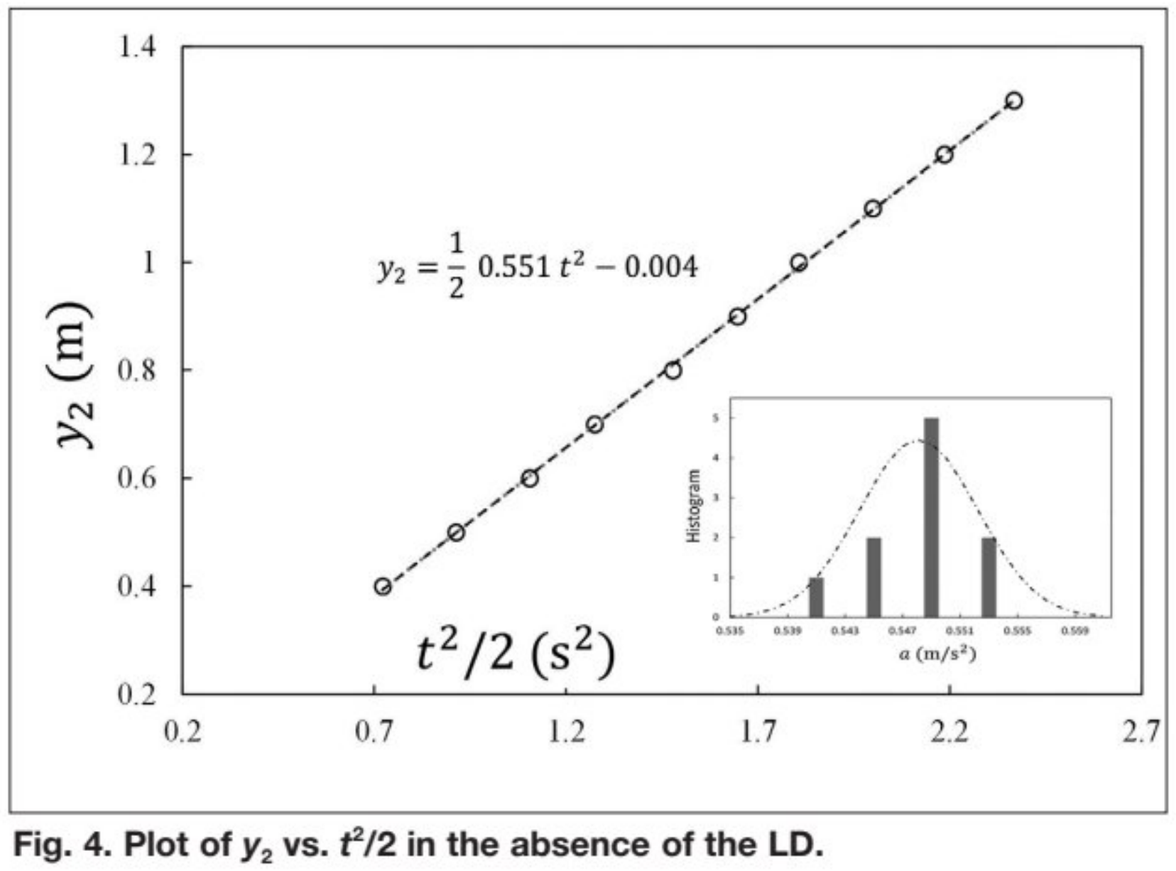

Fig. 4. Plot of y_2 vs. $t^2/2$ in the absence of the LD.

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Table II. Measured values of y_2 and t and results for the acceleration a, the uncertainty Δa, and the relative uncertainty $\Delta a/a$ in the absence of the LD for the machine moving at a constant acceleration (see Section A2 of the Appendix[9] for calculations).

i	1	2	3	4	5	6	7	8	9	10
y_2 (m)	0.400	0.500	0.600	0.700	0.800	0.900	1.000	1.100	1.200	1.300
t (s)	1.202	1.352	1.487	1.597	1.720	1.815	1.901	2.000	2.091	2.176
a (m/s^2)	0.554	0.547	0.543	0.549	0.541	0.546	0.553	0.550	0.549	0.549
Δa (m/s^2)	0.002	0.002	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001
$\Delta a/a$ (%)	0.4	0.3	0.3	0.2	0.2	0.2	0.2	0.2	0.2	0.2

Figure 4 shows the graph of y_2 in terms of $t^2/2$. Points of the graph are located almost along a straight line, which indicates movement with constant acceleration. The dashed line represents the best straight line that can fit the points of the graph. The slope of this line, i.e., 0.551 m/s^2, indicates the constant acceleration of the Atwood machine.

The histogram in the inset of Fig. 4 shows how many times each measured value of a is repeated, and the dashed curve shows the normal distribution with mean value $\bar{a} = 0.548$ m/s^2 and standard deviation $\sigma = 0.005$ m/s^2, which fits the histogram. Here, $n = 10$ is the total number of measurements, and the histogram graph tends to a normal distribution with increasing n. Therefore, we can consider the average value of uncertainties $\overline{\Delta a} = 0.002$ m/s^2 as the final uncertainty in measurement of a, which is in agreement with the standard uncertainty $\bar{\sigma} = \sigma/\sqrt{n} = 0.002$ m/s^2. The result of $a = 0.548 \pm 0.002$ m/s^2 shows the relative uncertainty in acceleration measurement equal to 0.4%, which is related to random errors. Therefore, the precision of measured acceleration in this experiment is 99.6%.

Accuracy of measured quantities

To determine the accuracy of the measured value of v, we use a semiempirical speed as the true value of v as follows: In the presence of the LD, before mass 1 arrives at the SR, we have $M_1 = M + \overline{m} + m + m_0$ and $M_2 = M - \overline{m}$ with $M = 77.45$ g, $m = 10.00$ g, and $m_0 = 5.185$ g. It is worth noting that in order to keep the speed of the Atwood machine constant over the distance y between the time sensors in Fig. 1, it is necessary to unbalance the values of mass 1 and mass 2 by a special value, which in the present work is equal to $\overline{m} = 0.83$ g. This unbalance or difference in mass 1 and mass 2 can be obtained experimentally by testing the machine at equilibrium so that it exactly lies at the threshold of motion. Therefore, the acceleration of mass 1 before it arrives at the SR is $a = 0.810$ m/s^2. The theoretical value of the speed of mass 1 just when it arrives at the SR is equal to $\sqrt{2ay_0} = 0.569$ m/s, which could be considered as the true value of v. Comparing this value with the measured value $\bar{v} = 0.567$ m/s yields the relative error of the measurement as –0.4%, which is related to systematic errors. The negative sign indicates that the measured value is smaller than the theoretical value. Therefore, the accuracy of the measured speed in this experiment is 99.6%. If we do not take into account the pulley's friction force F, then Eq. (3) yields $a = 0.932$ m/s^2, and the speed of mass 1 just when it arrives at the SR is equal to $\sqrt{2ay_0} = 0.611$ m/s; thus, its relative error would be –8%.

To determine the accuracy of the measured value of a, we use Eq. (3) with $M_1 = M + m$ and $M_2 = M$. This yields the theoretical value of acceleration of the machine as $a = 0.539$ m/s^2, which could be considered as the true value of a. Comparing this value with the measured value $\bar{a} = 0.548$ m/s^2 yields the relative error of the measurement as +2%, which is related to systematic errors. The positive sign indicates that the measured value is larger than the theoretical value. Therefore, the accuracy of the measured acceleration in this experiment is 98%. If we do not take into account the pulley's friction force F, then Eq. (3) yields $a = 0.665$ m/s^2; thus, its relative error would be –20%.

The level of confidence of the measurement result according to the use of normal distribution is 68%, which means that new measurements are obtained with a probability of 68% in the interval $(-\bar{\sigma}, +\bar{\sigma})$ around the average value. To increase the level of confidence, we can multiply the uncertainty by a coverage factor k. For $k = 2$ and 3, the level of confidence is equal to 95% and 99.7%, respectively. A better idea to increase the level of confidence is using the student's t distribution, which is suitable for small samples. Since we have $n = 10$ trials, we can multiply the uncertainty by a t score. According to the table of t distribution, for degree of freedom $n - 1 = 9$ and t score =2.262 and 4.024, the level of confidence is equal to 95% and 99.7%, respectively. Air friction has no noticeable effect on the measurement results, but incorrect readings of y and mass values and incorrect alignment of support column lead to decreasing the accuracy of the measurement results.

Summary and conclusion

Based on a detailed standard error analysis including both systematic and random errors (the accuracy and the precision) and the level of confidence, I experimentally demonstrated how FFPB affects measurements of acceleration and velocity in the Atwood machine. The uncertainty in the experimental results related to the random errors decreases as we increase the number of runs, so the precision of the results is improved. However, ignoring the FFPB is a major source of systematic errors and drastically decreases the accuracy of the measured quantities. I hope this paper provides physics instructors insight as they introduce undergraduates to comprehensive error analysis of experimental results.

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Acknowledgments

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8. G. O. Johnson, "Making Atwood's machine 'work,'" *Phys. Teach.* **39**, 154–158 (2001); see also Section A1 of the Appendix.[9]
9. Readers can access the Appendix at *TPT Online* at https://doi.org/10.1119/5.0211528, in the "Supplementary Material" section.
10. The theoretical value of the gravitational acceleration, $g = 9.797\ \text{m/s}^2$, has been obtained in the laboratory using the following formula: $g = A(1 + B\sin^2\theta - C\sin^2 2\theta)$, with $A = 9.780327$, $B = 0.0053024$, and $C = 0.0000058$, where I considered the latitude $\theta = 34.011°$ at the laboratory location.

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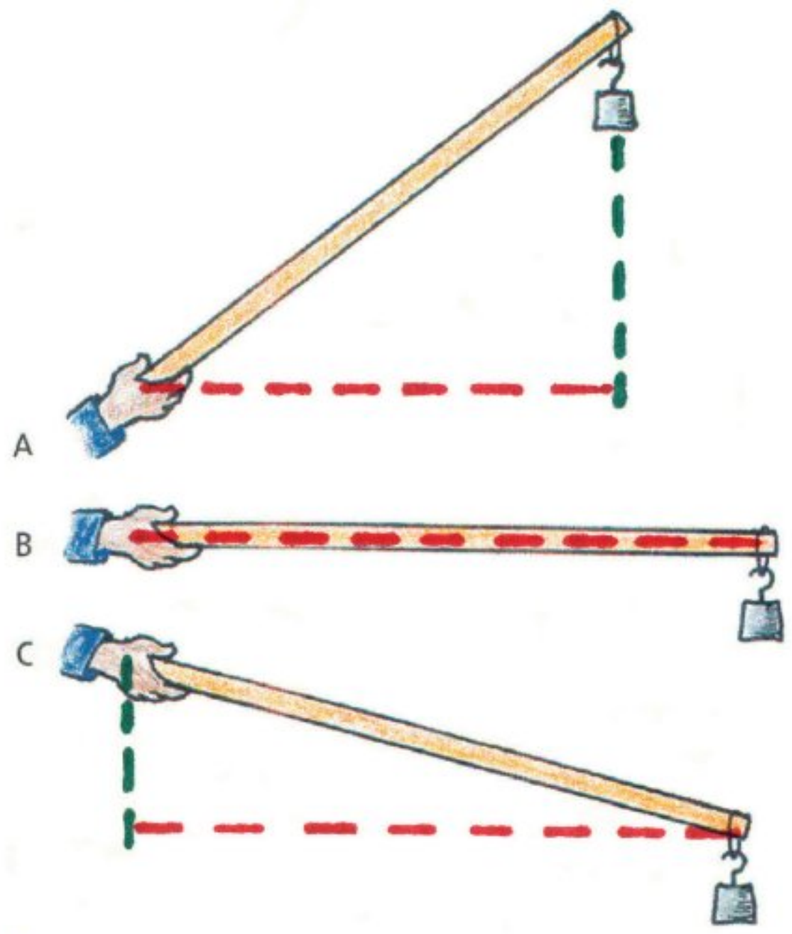
FIGURING PHYSICS

Phil Physicer holds a meterstick in three orientations as shown. He finds that holding it steady is easiest in

A. Position A B. Position B C. Position C
D. Any of these positions.



Answer: A

In each position, the grip of Phil's hand deals with the torque produced by stick with its dangling mass. That torque will be least where the lever arm (red dashed line) is shortest. That occurs for Position A. Holding the stick at the other positions involves greater torques; more for Position C, and most for Position B.

Try this and see! You'll find the least effort when the stick is vertical. Then you're playing with rotational inertia. All fun physics!

Mechanical equilibrium requires that both $\Sigma F = 0$ and $\Sigma \tau = 0$.

Figuring Physics April Answer

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